Combining Models and Observations: Bayesian Approaches

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Outline

- **Main Themes: Goals and Approaches**
- **Bayesian Hierachical Models**
- **Two Classes of Approaches with Examples**
- **Discussion**

Supported by NSF, EPA, NASA

Introduction

- Selected Motivations
  - computational & observational enhancements offer both new opportunities & new challenges
  - need for uncertainty management
- Goals:
  - develop probability distribution for unknowns of interest.
  - combine information: observations, theory, computer model output, past experience, etc.
- Framework: Bayesian Hierarchical Models

Bayesian Hierarchical Modeling (BHM)

- **BHM: sequence of conditional probability models**
- **Quintessential BHM: Data Y; Process of interest X**
  1. **Data Model** $[\ \mathbf{Y} \mid \mathbf{X}, \boldsymbol{\theta}\ ]$
  2. **Process Model** $[\ \mathbf{X} \mid \boldsymbol{\theta}]$
  3. **Parameter Model** $[\ \boldsymbol{\theta}\ ]$
- **Bayes' Theorem:** $[\ \mathbf{X}, \boldsymbol{\theta} \mid \mathbf{Y}\ ]$

Compare

- **"Statistics":** $[\ \mathbf{Y} \mid \boldsymbol{\theta}\ ]$ **(&** $[\ \boldsymbol{\theta}\ ]$ **for Bayesians)**
- **"Physics":** $[\ \mathbf{X} \mid \tilde{\boldsymbol{\theta}}(\mathbf{Y})\ ]$

Approaches

1. **Stochastic models incorporating science**

   (a) **Physical-statistical modeling** (*Berliner 2003 JGR*)
   From "F=ma" to process model [X | $\boldsymbol{\theta}$]
   Three examples

   (b) **Qualitative use of theory**
   (eg., Pacific SST model *Berliner et al. 2000 J. Climate*)

2. **Incorporating large-scale computer models**

   (a) **From model output to priors on**
   - **Parameters [$\boldsymbol{\theta}$]**
   - **Model output as samples from process model [X | $\boldsymbol{\theta}$]**

   (b) **Model output as "observations" (Y)**

3. **Combinations**

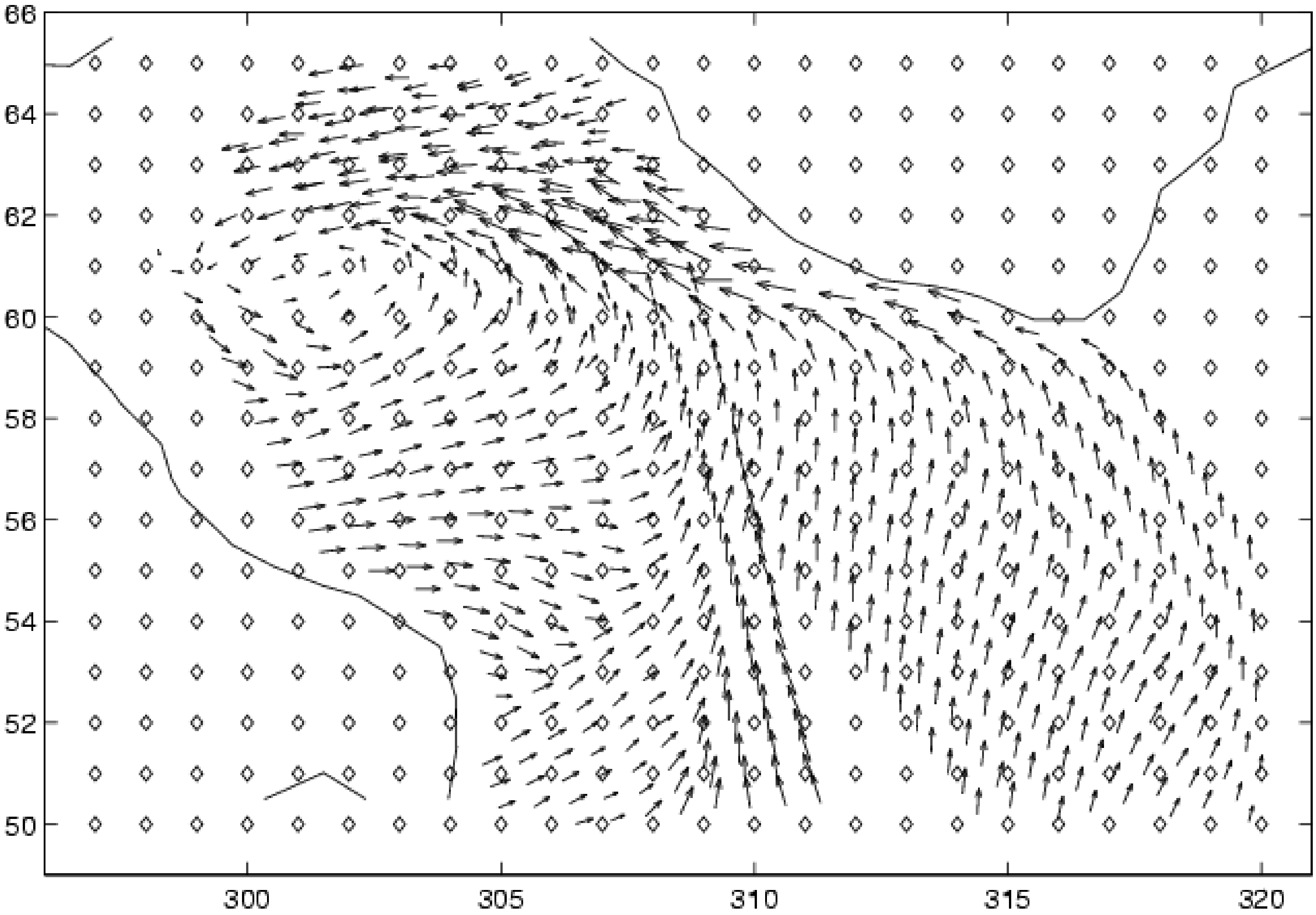
Lab-Sea Air Model *Royle et al 1998*

- **Process: near-surface (10m) winds** $\mathbf{W} = (\mathbf{U}, \mathbf{V})$
- **Why? Several uses (e.g., driving ocean models)**
- **Data: Scatterometer-based estimates**
- **Physics: Geostrophic Approximation "Winds are linear in the gradient of the pressure field"**

$$\mathbf{v_g} = \mathbf{c}\frac{\partial \mathbf{P}}{\partial \mathbf{x}}, \ \mathbf{u_g} = -\mathbf{c}\frac{\partial \mathbf{P}}{\partial \mathbf{y}}$$

Balance pressure potential & Coriolis
Pretty good at mid-latitudes & upper altitude
Not good at 10m (friction, turbulence); or if large curvature in pressure field

Lab Sea Grid and Scatterometer Data



Stochastic Geostrophic Model

Let $(\mathbf{U}, \mathbf{V}), \mathbf{P}$ be gridded wind vector components and pressure.

- **Data Model:** $[\mathbf{D_u}, \mathbf{D_v} | \mathbf{U}, \mathbf{V}, \sigma^2_{\mathbf{d}}]$:

$$\begin{pmatrix} \mathbf{D_u} \\ \mathbf{D_v} \end{pmatrix} \sim \mathbf{Gau}\left(\mathbf{K}\begin{pmatrix} \mathbf{U} \\ \mathbf{V} \end{pmatrix}, \begin{pmatrix} \boldsymbol{\sigma}^2_{\mathbf{d}}\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\sigma}^2_{\mathbf{d}}\mathbf{I} \end{pmatrix}\right)$$

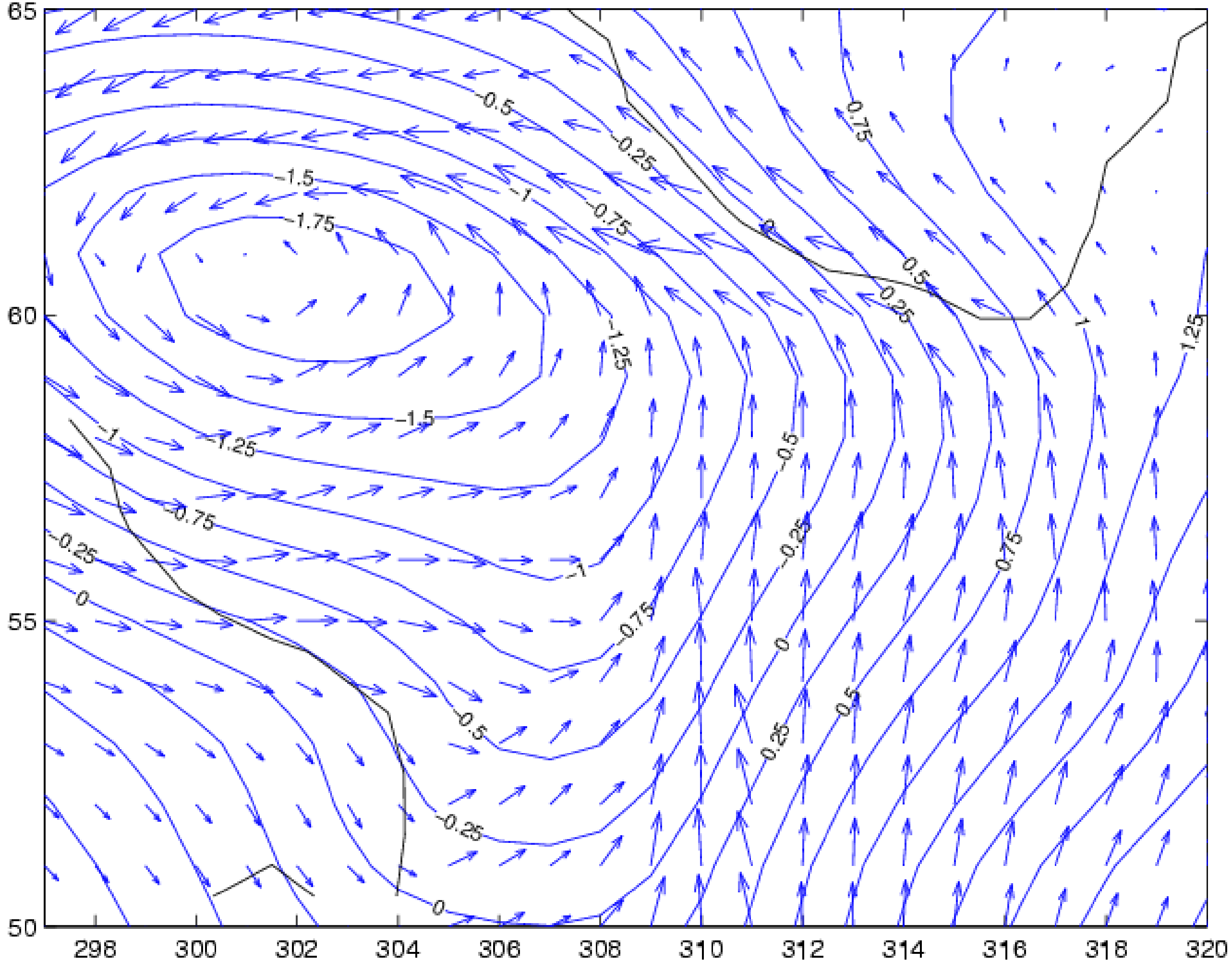
- **Process Model:**
  - $[\mathbf{U}, \mathbf{V} | \mathbf{P}, \boldsymbol{\mu}_{\mathbf{u}}, \boldsymbol{\mu}_{\mathbf{v}}, \boldsymbol{\beta}, \boldsymbol{\Sigma}_{\mathbf{uv}}]$:

$$\begin{pmatrix} \mathbf{U} \\ \mathbf{V} \end{pmatrix} \sim \mathbf{Gau}\left(\begin{pmatrix} \boldsymbol{\mu}_{\mathbf{u}}\mathbf{1} + \mathbf{B_u}(\boldsymbol{\beta})\mathbf{P} \\ \boldsymbol{\mu}_{\mathbf{v}}\mathbf{1} + \mathbf{B_v}(\boldsymbol{\beta})\mathbf{P} \end{pmatrix}, \boldsymbol{\Sigma}_{\mathbf{uv}} \otimes \mathbf{I}^*\right)$$

    B's: discrete derivative estimates with random coefficients
  - $[\mathbf{P} | \boldsymbol{\mu}_{\mathbf{p}}, \boldsymbol{\Sigma}_{\mathbf{p}}]$ **(Thiebaux 1985)**

- **Parameter priors**

Posterior Means: Winds and Pressure Field



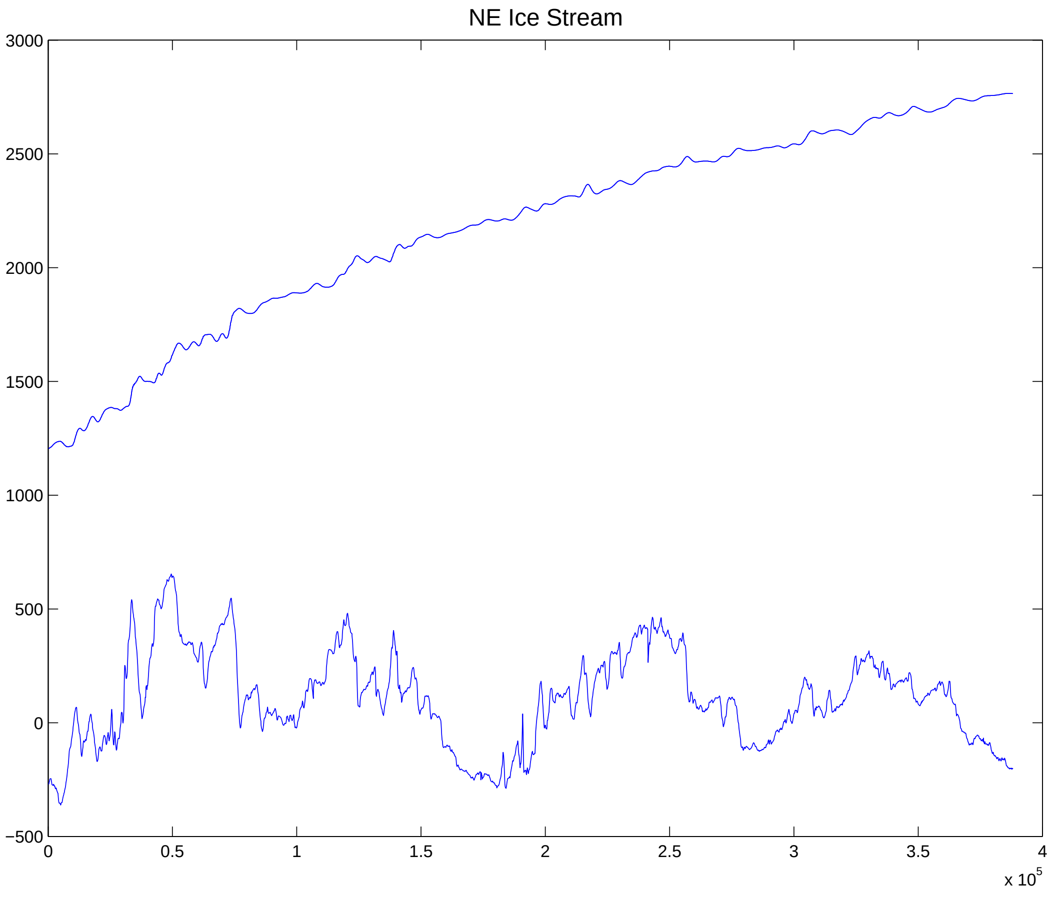
Glacial Dynamics *Berliner et al. 2008 J. Glaciol.*

- **Flow: gravity moderated by drag (base and sides) & ....**
- **Simple flow models: flow from geometry.**

Data

Program for Arctic Climate Regional Assessments (PARCA)
Radarsat Antarctic Mapping Project (RAMP)

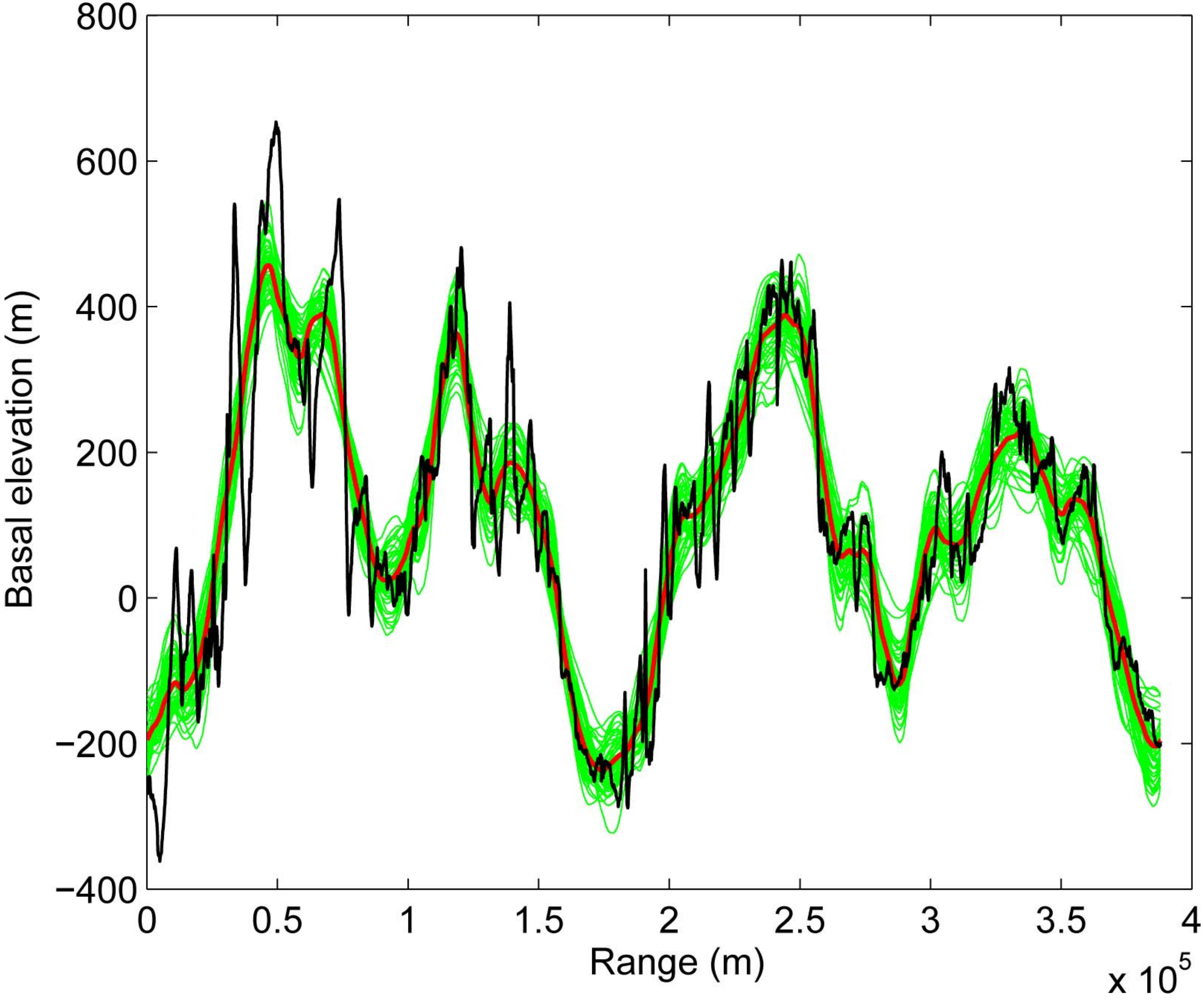
- **S: surface topography (Laser altimetry)**
- **B: basal topography (Radar altimetry)**
- **U: velocity data (Interferometry)**



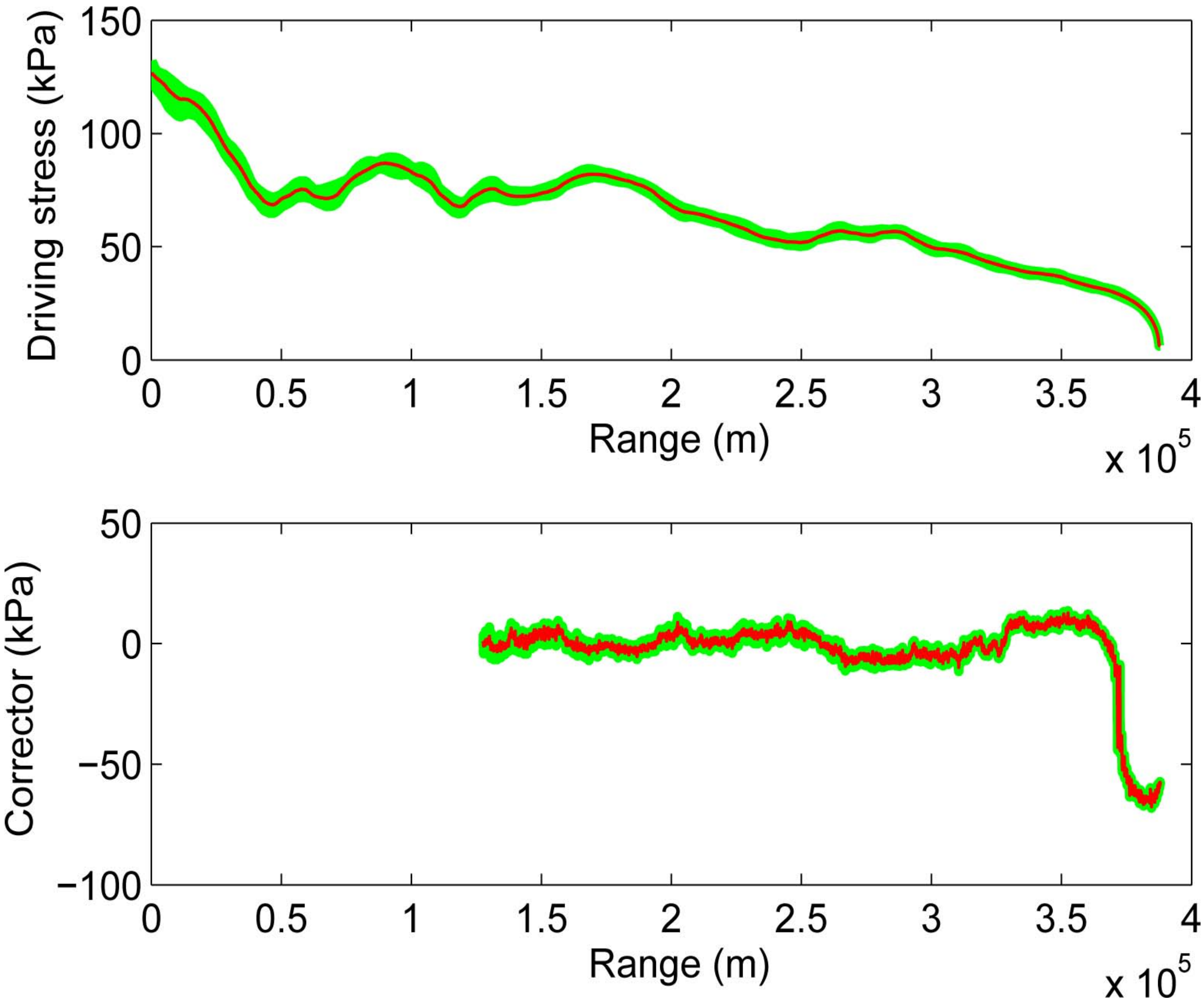
NE Ice Stream
3000
2500
2000
1500
1000
500
0
−500
0
0.5
1
1.5
2
2.5
3
3.5
4
x 10^5

Modelling

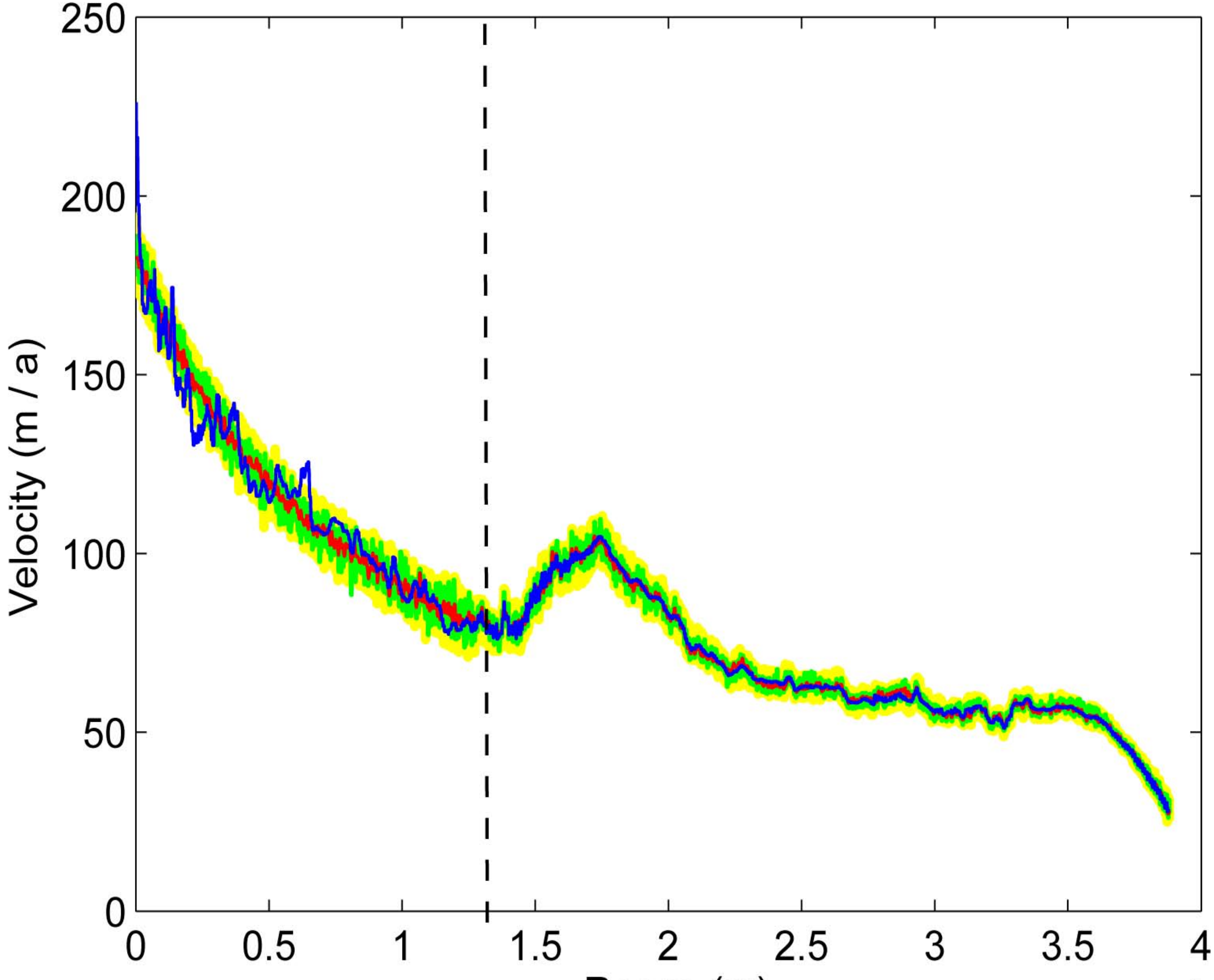
- Processes: surface; s: base; H: thickness; u: velocity
- Physical Model
  - Basal Stress $\boldsymbol{\tau} = -\boldsymbol{\rho}\, \mathrm{g}\, \mathrm{H}\, \mathrm{s}' + \mathrm{stuff}$
  - Velocities $\mathrm{u} = \mathrm{u_b} + \boldsymbol{\beta}_0\, \mathrm{H}\, \boldsymbol{\tau}^{\mathrm{n}}$
    where $\mathrm{u_b} = \mathrm{k}\, \boldsymbol{\tau}^{\mathrm{p}}\, (\boldsymbol{\rho}\, \mathrm{g}\, \mathrm{H})^{-\mathrm{q}}$
- Our Model
  - Basal Stress $\tilde{\boldsymbol{\tau}} = -\boldsymbol{\rho}\, \mathrm{g}\, \tilde{\mathrm{H}}\, \tilde{\mathrm{s}}' + \boldsymbol{\eta}$
    where $\boldsymbol{\eta}$ is a "corrector process", $\tilde{\mathrm{H}}, \tilde{\mathrm{s}}$ are unknown
  - Velocities $\mathrm{u} = \tilde{\mathrm{u}}_\mathrm{b} + \boldsymbol{\beta}\, \tilde{\mathrm{H}}\, \tilde{\boldsymbol{\tau}}^{\mathrm{n}} + \mathrm{e}$
    where $\mathrm{u_b} = \mathrm{k}\, \tilde{\boldsymbol{\tau}}^{\mathrm{p}}\, (\boldsymbol{\rho}\, \mathrm{g}\, \tilde{\mathrm{H}})^{-\mathrm{q}}$ or an unknown constant,
    $\boldsymbol{\beta}$ is unknown, e is a noise process.
  - Smoothing



800
600
400
200
0
−200
−400
Basal elevation (m)
0
0.5
1
1.5
2
2.5
3
3.5
4
Range (m)
x 10^5



Driving stress (kPa)
150
100
50
0
0
0.5
1
1.5
2
2.5
3
3.5
4
Range (m)
x 10^5
Corrector (kPa)
50
0
−50
−100
0
0.5
1
1.5
2
2.5
3
3.5
4
Range (m)
x 10^5



250
200
150
100
50
0
Velocity (m / a)
0
0.5
1
1.5
2
2.5
3
3.5
4
Range (m)
x 10^5

Air-Sea Interaction *Berliner et al 2004 JGR*

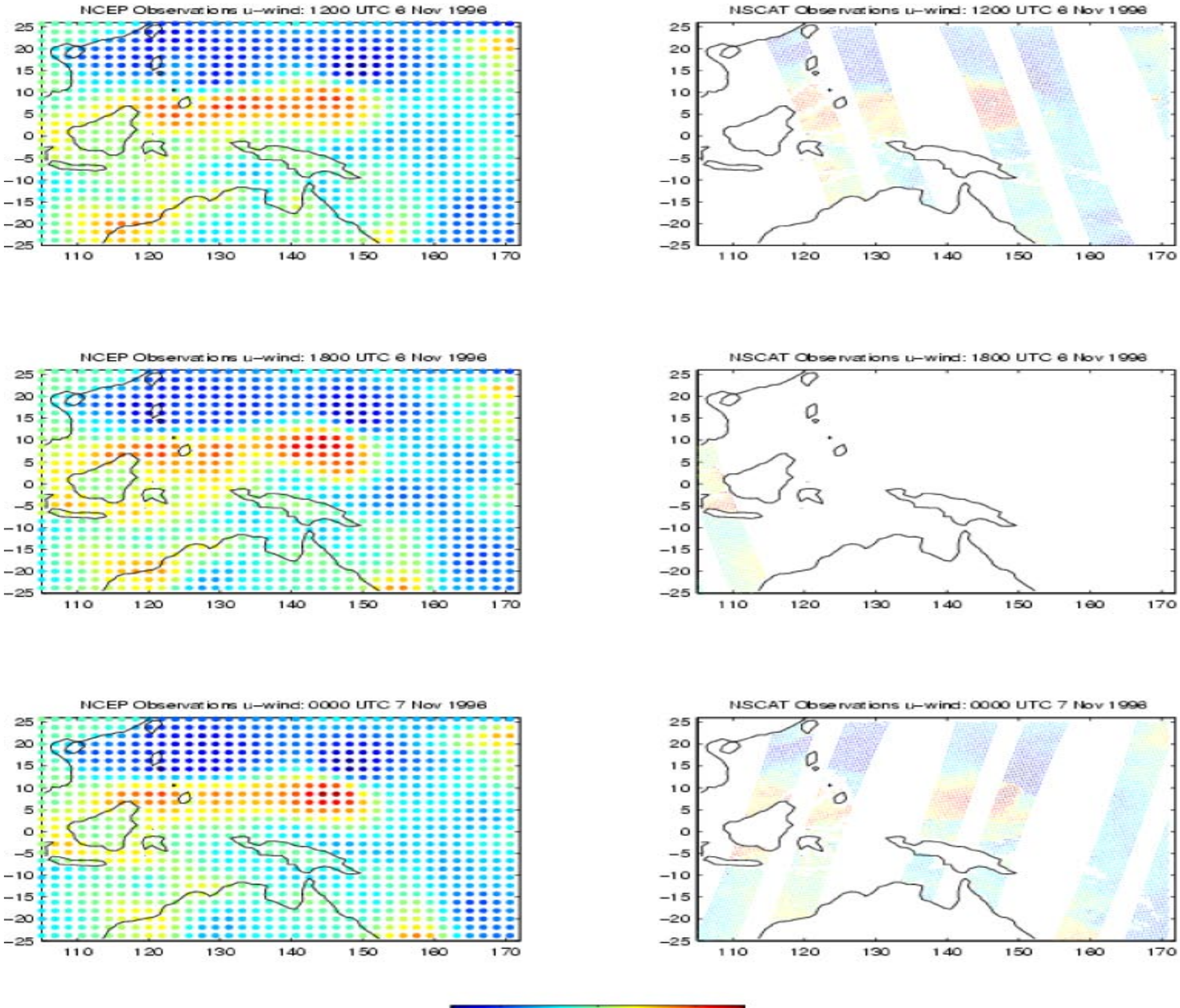
- **Processes:**
  - **Ocean streamfunction ψ (feature related to currents)**
  - **Near-surface winds W**
- **Data**
  - **D_a Wind data (scatterometer)**
  - **D_o Ocean data (altimeter)**
- **Physics: Quasi-geostrophy (QG)**

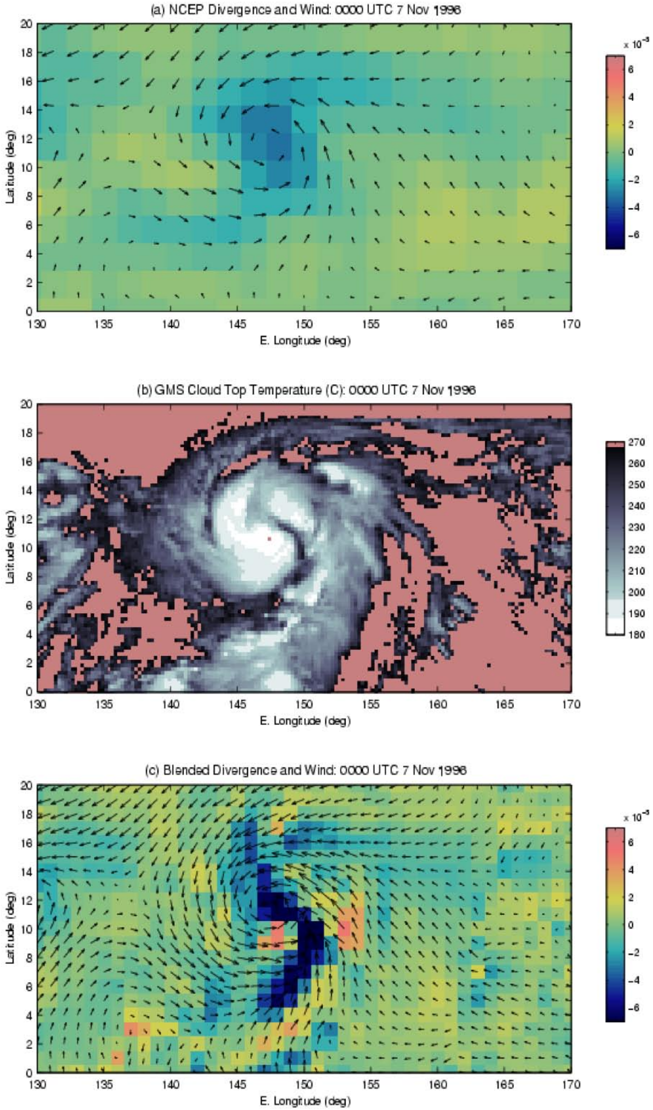
$$(\nabla^2 - \frac{1}{r^2})\frac{\partial \psi}{\partial t} = -J(\psi, \nabla^2\psi) - \beta\frac{\partial \psi}{\partial x} + \frac{1}{\rho H}\,\mathrm{curl}_z \tau(W) - \gamma\nabla^2\psi + a_h\nabla^4\psi$$

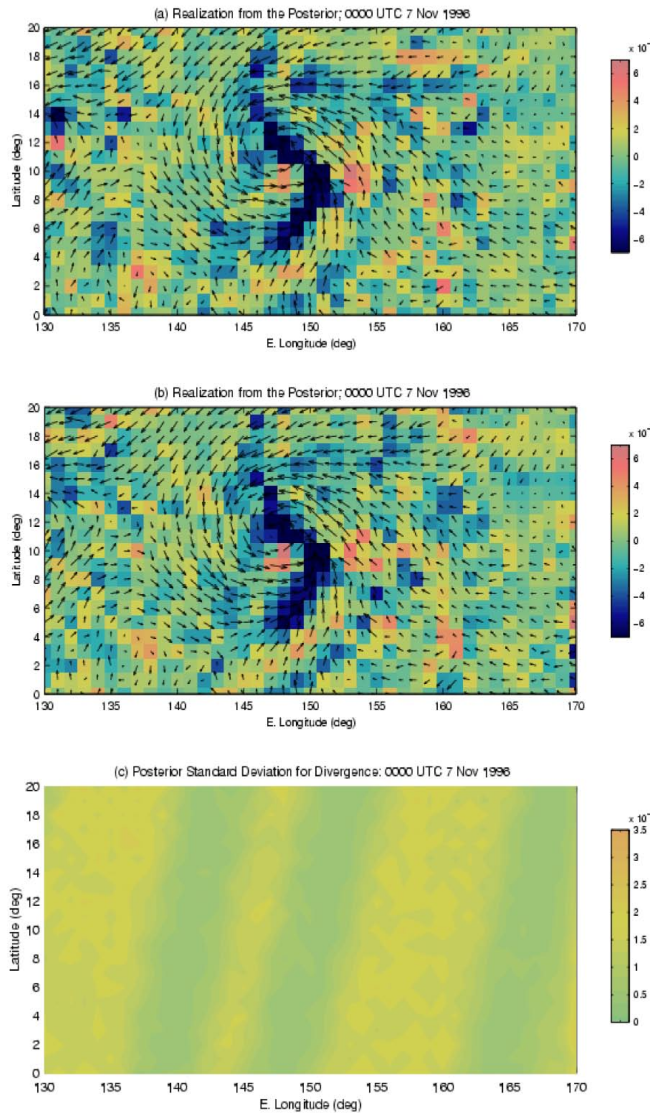
- **Stochastic: $[\psi|W]$ from QG**
  Couple with $[W]$ and we're "done"

Near-surface Ocean Winds *Wikle et al 2001 JASA*

- **Data sources**
  - **Scatterometer**
  - **NCEP Analyses**
- **Space-time process model**
  - **Modes of linearized shallow-fluid equations (large scales)**
  - **Wavelets (small scales)**
  - **Both with time-varying coefficients**
- **Priors: turbulence theory**


NCEP Observations u-wind: 1200 UTC 6 Nov 1996
NSCAT Observations u-wind: 1200 UTC 6 Nov 1996
NCEP Observations u-wind: 1800 UTC 6 Nov 1996
NSCAT Observations u-wind: 1800 UTC 6 Nov 1996
NCEP Observations u-wind: 0000 UTC 7 Nov 1996
NSCAT Observations u-wind: 0000 UTC 7 Nov 1996


(a) NCEP Divergence and Wind: 0000 UTC 7 Nov 1996
(b) GMS Cloud Top Temperature (C): 0000 UTC 7 Nov 1996
(c) Blended Divergence and Wind: 0000 UTC 7 Nov 1996
Latitude (deg)
E. Longitude (deg)
x 10^-5


(a) Realization from the Posterior; 0000 UTC 7 Nov 1996
(b) Realization from the Posterior; 0000 UTC 7 Nov 1996
(c) Posterior Standard Deviation for Divergence: 0000 UTC 7 Nov 1996
Latitude (deg)
E. Longitude (deg)
x 10^-5

Discussion

- No claim of solving PDE
  - we often introduce noise (but not "solving" SPDE)
  - made parameters random
  - role of stability (i.e., CFL conditions) depends on data quality and goals
- No free lunch: Concerns about
  - computation (MCMC; importance sampling)
  - quality of each component of a BHM
- Transition to Part II: We usually need very large ensembles
  Not practical if $[\ \mathbf{X} \mid \boldsymbol{\theta}\ ]$ involves a massive computer model

Combining Models and Observations: Bayesian Approaches II

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Goals

- Develop probability distribution for unknowns of interest.
- Combine information: observations, theory, computer model output, past experience, etc.
- Do all this while accounting for uncertainty
- Framework: Bayesian Hierarchical Models

Supported by NSF, EPA, NASA

Bayesian Hierarchical Modeling (BHM)

- **BHM: sequence of conditional probability models**
- **Quintessential BHM: Data Y; Process of interest X**
  1. **<u>Data Model</u>** $[\ \mathbf{Y} \mid \mathbf{X}, \boldsymbol{\theta}\ ]$
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  3. **<u>Parameter Model</u>** $[\ \boldsymbol{\theta}\ ]$
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Compare

- **"Statistics":** $[\ \mathbf{Y} \mid \boldsymbol{\theta}\ ]$ **(&** $[\ \boldsymbol{\theta}\ ]$ **for Bayesians)**
- **"Physics":** $[\ \mathbf{X} \mid \tilde{\boldsymbol{\theta}}(\mathbf{Y})\ ]$

Approaches

1. **Stochastic models incorporating science**
   (a) **Physical-statistical modeling** (*Berliner 2003 JGR*)
   From "F=ma" to process model [**X** | $\boldsymbol{\theta}$]
   (Three examples)
   (b) **Qualitative use of theory**
   (eg., Simple Pacific SST model *Berliner et al. 2000 J. Climate*)
2. **Incorporating large-scale computer models**
   (a) **From model output to priors on**
   - **Parameters** [$\boldsymbol{\theta}$]
   - **Model output as samples from process model** [**X** | $\boldsymbol{\theta}$]

   (b) **Model output as observations**
3. **Combinations**

(a) From model output to priors

- Think of model output runs $\mathbf{O_1, \cdots, O_n}$ as a sample from some distribution
- Do data analysis on the O's to estimate distribution
  - develop prior on X: [X] or [X| $\boldsymbol{\theta}$]
  - develop [$\boldsymbol{\theta}$]
- Common Example: O's are spatial fields: estimate spatial covariance function of X based on O's.

Ex) Anthropogenic Climate Change

- **Detection & Attribution: CO_2 and temperature**
- **g spatial pattern of anticipated CO_2 impacts (usually based on a climate system model)**
- **Model: Data = a g + noise**
- **Test a = 0 vrs a = μ_c**

BHM *Berliner, Levine & Shea 2000 J. Climate 2000*

- **NCAR Climate System Model (CSM)**

- **1000 year control run; 300 year CO_2-forced run**
- **Data: Jones' surface temperature record**

Process: True Surface Temperature T Record

1. $[\mathbf{Data} \mid \mathbf{T}] : \mathbf{D} \sim \mathbf{Gau}(\mathbf{KT}, \boldsymbol{\Sigma}_\mathbf{D})$

   K maps data to CSM grid; $\boldsymbol{\Sigma}_\mathbf{D}$ from literature.

2. $[\mathbf{T} \mid \mathbf{a}]$: $\mathbf{T} \sim \mathbf{Gau}(\mathbf{aG}, \boldsymbol{\Sigma}_\mathbf{T})$

3. $[\mathbf{a}] = \mathbf{p}\,\mathbf{Gau}(\mathbf{0}, \boldsymbol{\tau}^2) + (\mathbf{1} - \mathbf{p})\,\mathbf{Gau}(\boldsymbol{\mu}_\mathbf{c}, \boldsymbol{\tau}_\mathbf{c}^2)$

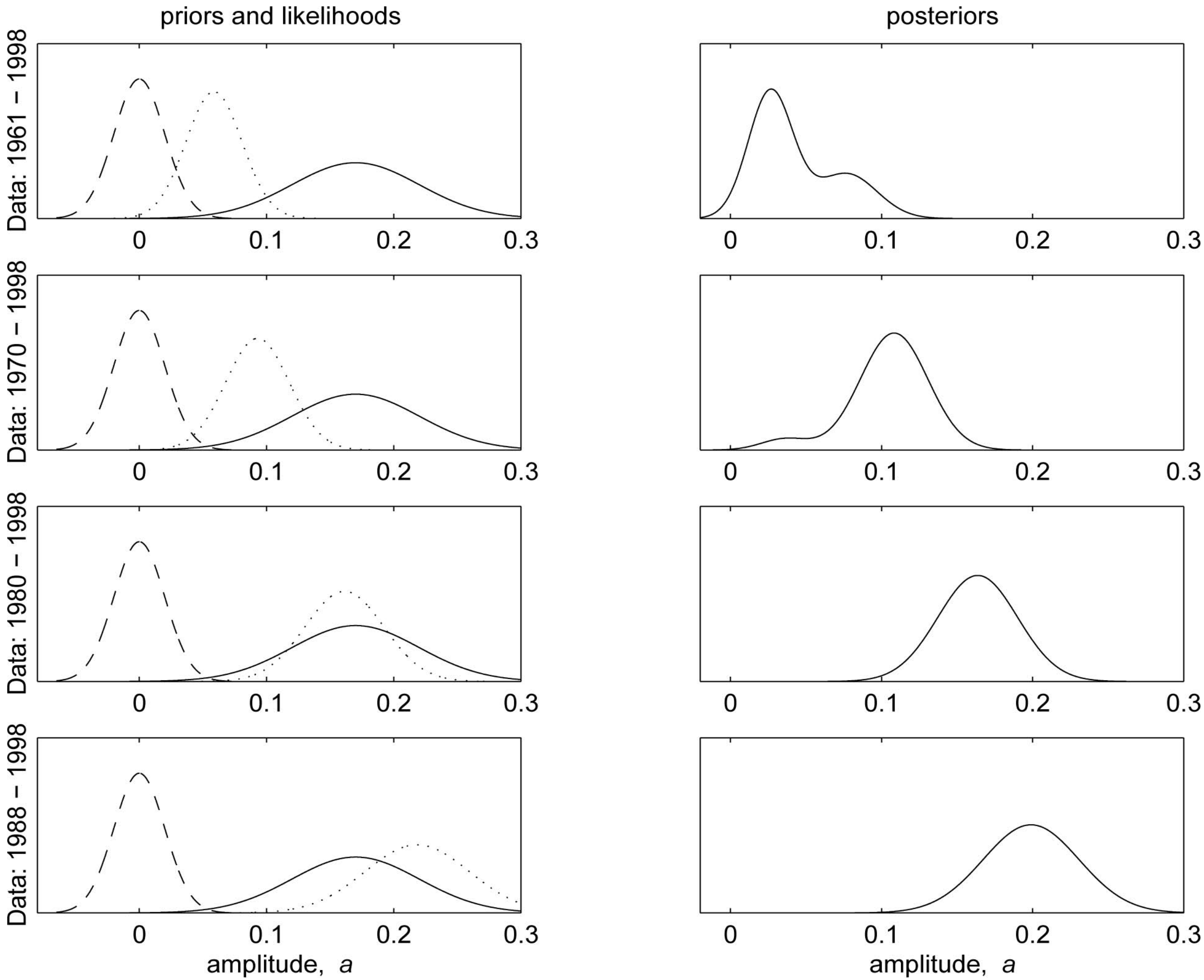
- **$\boldsymbol{\Sigma}_\mathbf{T}$: estimated using the model output.**
- **g: (CO_2-forced output) minus (control output).**
- **Hyperparameter estimation via subsampling model output.**
  - **Control Run: broken into 30 samples of length 10.**
    Regress sample onto g.
    Produces 30 estimates of a under "no forcing"
    Use their variance to estimate $\boldsymbol{\tau}^2$
  - **Forced Run: similar procedure to estimate $\boldsymbol{\mu}_c, \boldsymbol{\tau}_c^2$**

Analyses

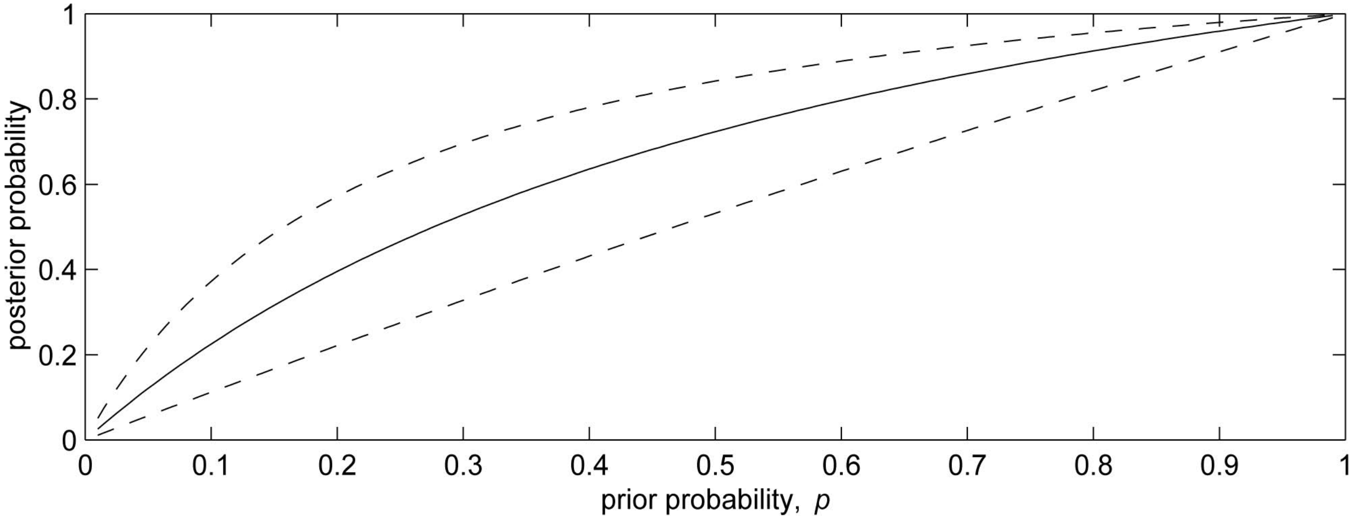
- **Let $\hat{\mathbf{a}}$ be generalized least squares estimate of $\mathbf{a}$: Posterior is**

$$[\mathbf{a} \mid \hat{\mathbf{a}}] = \mathbf{p}(\hat{\mathbf{a}})\,\mathbf{Gau}(\cdot,\cdot) + (\mathbf{1} - \mathbf{p}(\hat{\mathbf{a}}))\,\mathbf{Gau}(\cdot,\cdot)$$

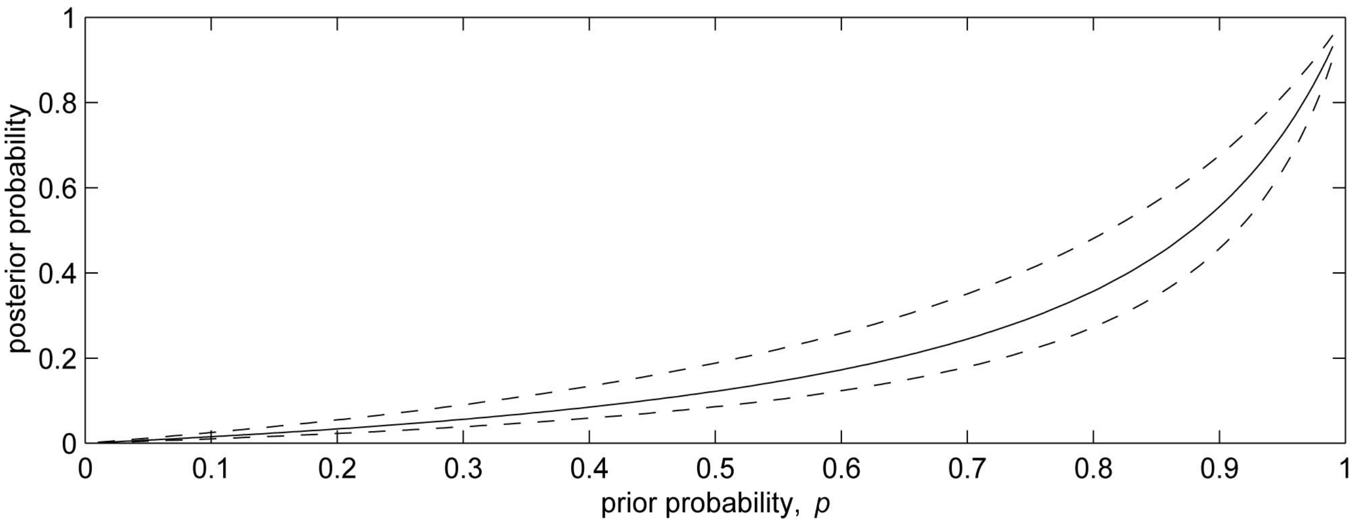
- **Uncertainty about uncertainty:**
  ranges over classes of priors and as p varies
- $\mathbf{P}(\mathbf{a} \approx \mathbf{0} \mid \hat{\mathbf{a}})$ **&** $\mathbf{P}(\mathbf{a} \approx \mu_{\mathbf{c}} \mid \hat{\mathbf{a}})$



priors and likelihoods
posteriors
Data: 1961 – 1998
Data: 1970 – 1998
Data: 1980 – 1998
Data: 1988 – 1998
0
0.1
0.2
0.3
amplitude, a
amplitude, a



posterior probability
prior probability, p
0
0.2
0.4
0.6
0.8
1
0.1
0.3
0.5
0.7
0.9



posterior probability
prior probability, p
0
0.2
0.4
0.6
0.8
1
0.1
0.3
0.5
0.7
0.9

(b) Model output as observations

- **Act as if no <u>formal</u> difference between model output & observations**
- **In many cases "observations" are "model output"**
- **Nice way to combine information sources**
  "Observe" what you can; compute what you can't"
- **Experimental Design**
  Combined observational-computer model experiments
- **Other contexts!! (weather forecasting)**

Multimodel ensembles as observations

- **Set-up:** $\mathbf{m = 1, \ldots, M}$ **models.**
  Scalar (for now) climate variable X. (time fixed)
- **Data Model: Three Main Steps:**

For $\mathbf{k^{th}}$ ensemble member from Model m:

$$\begin{aligned}
\mathbf{Y_{mk}} &= \boldsymbol{\mu}_{\mathbf{m}} + \mathbf{e_{mk}}, \ \textbf{(Step 1)} \\
&= (\boldsymbol{\beta} + \mathbf{b_m} + \mathbf{e}_{\boldsymbol{\mu}_{\mathbf{m}}}) + \mathbf{e_{mk}}, \ \textbf{(Step 2)} \\
&= ((\mathbf{X} + \mathbf{e}_{\boldsymbol{\beta}}) + (\mathbf{b_{0m}} + \mathbf{e_{b_m}}) + \mathbf{e}_{\boldsymbol{\mu}_{\mathbf{m}}}) + \mathbf{e_{mk}}, \ \textbf{(Step 3)}
\end{aligned}$$

- **X vrs $\boldsymbol{\beta}$ is key**

Formally:

$\vec{\mathbf{Y}}_{\mathrm{m}}$: **ensemble of size $\mathrm{n_m}$ of derived estimates of X from model m.**

1. **Given means and variances $\boldsymbol{\mu}_{\mathrm{m}}, \boldsymbol{\sigma}^2_{\mathrm{Y_m}}$;**
   $\vec{\mathbf{Y}}_{\mathrm{m}}$ are independent and

$$\vec{\mathbf{Y}}_{\mathrm{m}} | \boldsymbol{\mu}_{\mathrm{m}} \sim \mathbf{Gau}(\boldsymbol{\mu}_{\mathrm{m}} \vec{\mathbf{1}}_{\mathrm{n_m}}, \boldsymbol{\sigma}^2_{\mathrm{Y_m}} \ \mathbf{I}_{\mathrm{n_m}})$$

2. **Given $\boldsymbol{\beta}$, biases $\mathrm{b_m}$ and variances $\boldsymbol{\sigma}^2_{\boldsymbol{\mu}_{\mathrm{m}}}$;**
   $\boldsymbol{\mu}_{\mathrm{m}}$ are independent and

$$\boldsymbol{\mu}_{\mathrm{m}} | \boldsymbol{\beta}, \mathbf{b}_{\mathrm{m}} \sim \mathbf{Gau}(\boldsymbol{\beta} + \mathbf{b}_{\mathrm{m}}, \ \boldsymbol{\sigma}^2_{\boldsymbol{\mu}_{\mathrm{m}}})$$

3. **Given X,**

$$\boldsymbol{\beta} | \mathbf{X} \sim \mathbf{Gau}(\mathbf{X}, \ \boldsymbol{\sigma}^2_{\boldsymbol{\beta}}) \text{ and } \mathbf{b}_{\mathrm{m}} | \mathbf{X} \sim \mathbf{Gau}(\mathbf{b}_{0\mathrm{m}}, \boldsymbol{\sigma}^2_{\mathrm{b_m}})$$

Implied Marginal: "Y given X"

Integrating out β induces dependence:

$$\begin{pmatrix} \vec{\mathbf{Y}}_1 \\ \vec{\mathbf{Y}}_2 \\ \vdots \\ \vec{\mathbf{Y}}_M \end{pmatrix} | \mathbf{X} \sim \mathrm{Gau}\left(\begin{pmatrix} (\mathbf{X} + b_{01})\vec{\mathbf{1}}_{n_1} \\ (\mathbf{X} + b_{02})\vec{\mathbf{1}}_{n_2} \\ \vdots \\ (\mathbf{X} + b_{0M})\vec{\mathbf{1}}_{n_M} \end{pmatrix}, \begin{pmatrix} \Sigma_1 & C_{12} & \ldots & C_{1M} \\ C_{21} & \Sigma_2 & \ldots & C_{2M} \\ \vdots & & & \vdots \\ C_{M1} & \ldots & \ldots & \Sigma_M \end{pmatrix} \right),$$

- $C_{mm'}$ is $n_m \times n_{m'}$ with all entries σ^2_{β}
- $v^2_m = \sigma^2_{\mu_m} + \sigma^2_{b_m}$ and

$$\Sigma_m = \begin{pmatrix} \sigma^2_{\beta} + v^2_m + \sigma^2_{Y_m} & \sigma^2_{\beta} + v^2_m & \ldots & \ldots \\ \sigma^2_{\beta} + v^2_m & \sigma^2_{\beta} + v^2_m + \sigma^2_{Y_m} & \ldots & \ldots \\ \vdots & & & \vdots \\ \ldots & \ldots & \ldots & \sigma^2_{\beta} + v^2_m + \sigma^2_{Y_m} \end{pmatrix}$$

Remarks

- **Covariances in marginal**
  - **Modify intuition about value of increasing ensemble size**
  - **Infinite ensembles do not give "perfect" forecasts: if all biases are 0, "infinite" ensembles tell the value of $\boldsymbol{\beta}$, not X**
- **d dimensional X: $\boldsymbol{\sigma}$'s become Σ's**

Hemispheric Surface Temperatures

- d=2. X: hemispheric- & monthly-averaged surface temp's
- Observations Y: 1882-2001. Model output O: 2002-2097.
- M=2: PCM, CCSM (THANKS: Claudia Tebaldi, NCAR)

Background

- Anthropogenic Climate Change:
  - CO_2 emissions forecasts: IPCC-SRES scenarios (we used 3)
  - Plugged into models: Climate forecasts
- Our view here
  - Climate-weather: multiscale phenomena
  - "Climate" as parameters of distribution of "weather" (*Berliner 2003: Stat. Sci.*)

Model Overview

1. $[\mathbf{Y}|\mathbf{X}^{\mathrm{p}}, \boldsymbol{\theta}][\mathbf{O}|\mathbf{X}^{\mathrm{f}}, \boldsymbol{\theta}]$
   - $[\mathbf{Y}|\mathbf{X}^{\mathrm{p}}, \boldsymbol{\theta}]$: **measurement error model**
   - $[\mathbf{O}|\mathbf{X}^{\mathrm{f}}, \boldsymbol{\theta}]$: **3 stage data model above, with**
     - **conditional independence of O over time.**
     - **covariances** $\boldsymbol{\Sigma}_{\boldsymbol{\beta}}, \boldsymbol{\Sigma}_{\boldsymbol{\mu}_{\mathrm{m}}}, \boldsymbol{\Sigma}_{\mathbf{Y}_{\mathrm{m}}}$ **and biases constant**
     - $\boldsymbol{\Sigma}_{\boldsymbol{\mu}_{\mathrm{m}}} + \boldsymbol{\Sigma}_{\mathbf{Y}_{\mathrm{m}}} = \boldsymbol{\Sigma}_{\mathrm{m}}$

2. $[\mathbf{X}^{\mathrm{p}}|\boldsymbol{\theta}][\mathbf{X}^{\mathrm{f}}|\mathbf{X}^{\mathrm{p}},\boldsymbol{\theta}]$

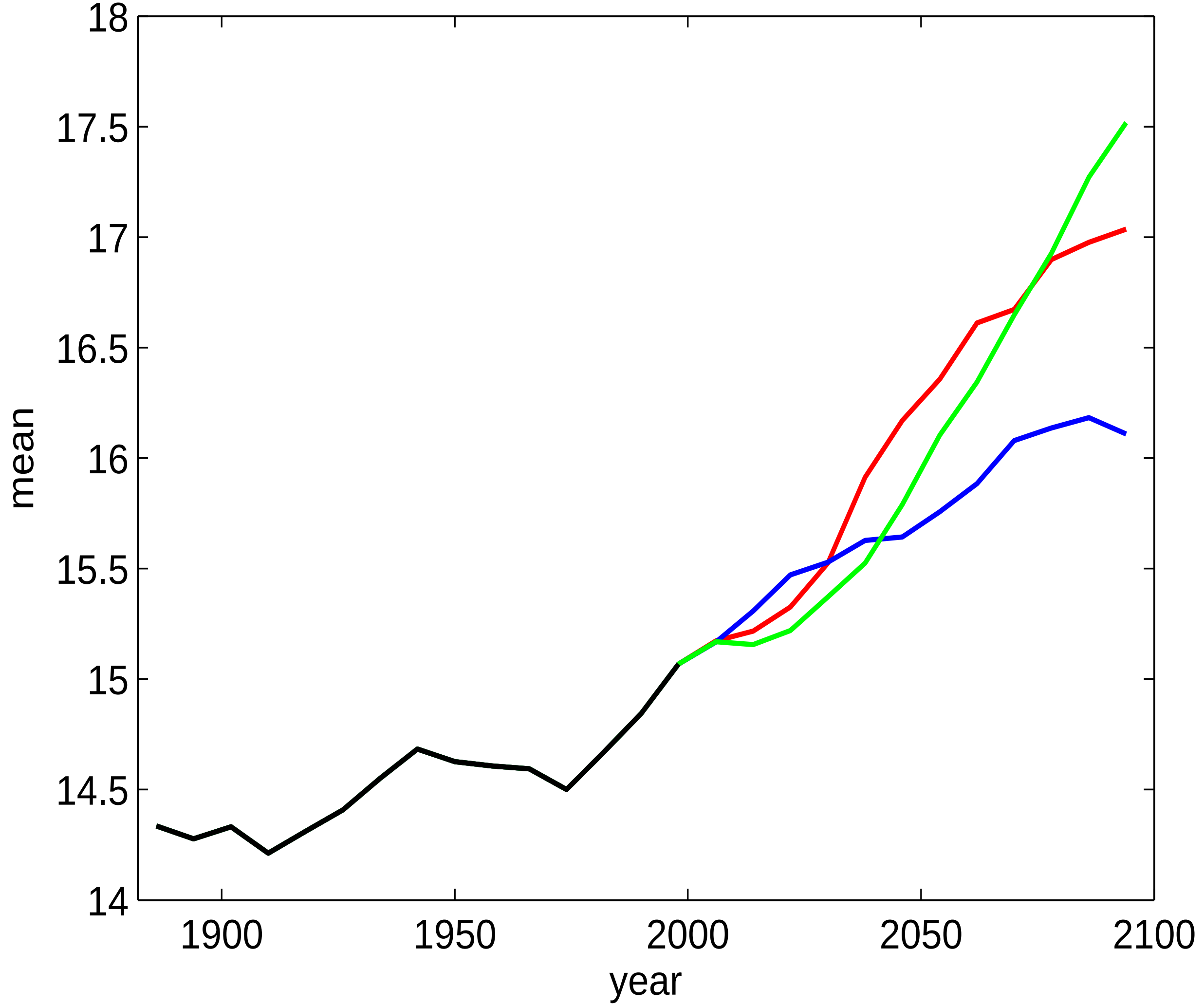
- **Time series models (AR) with time varying parameters**

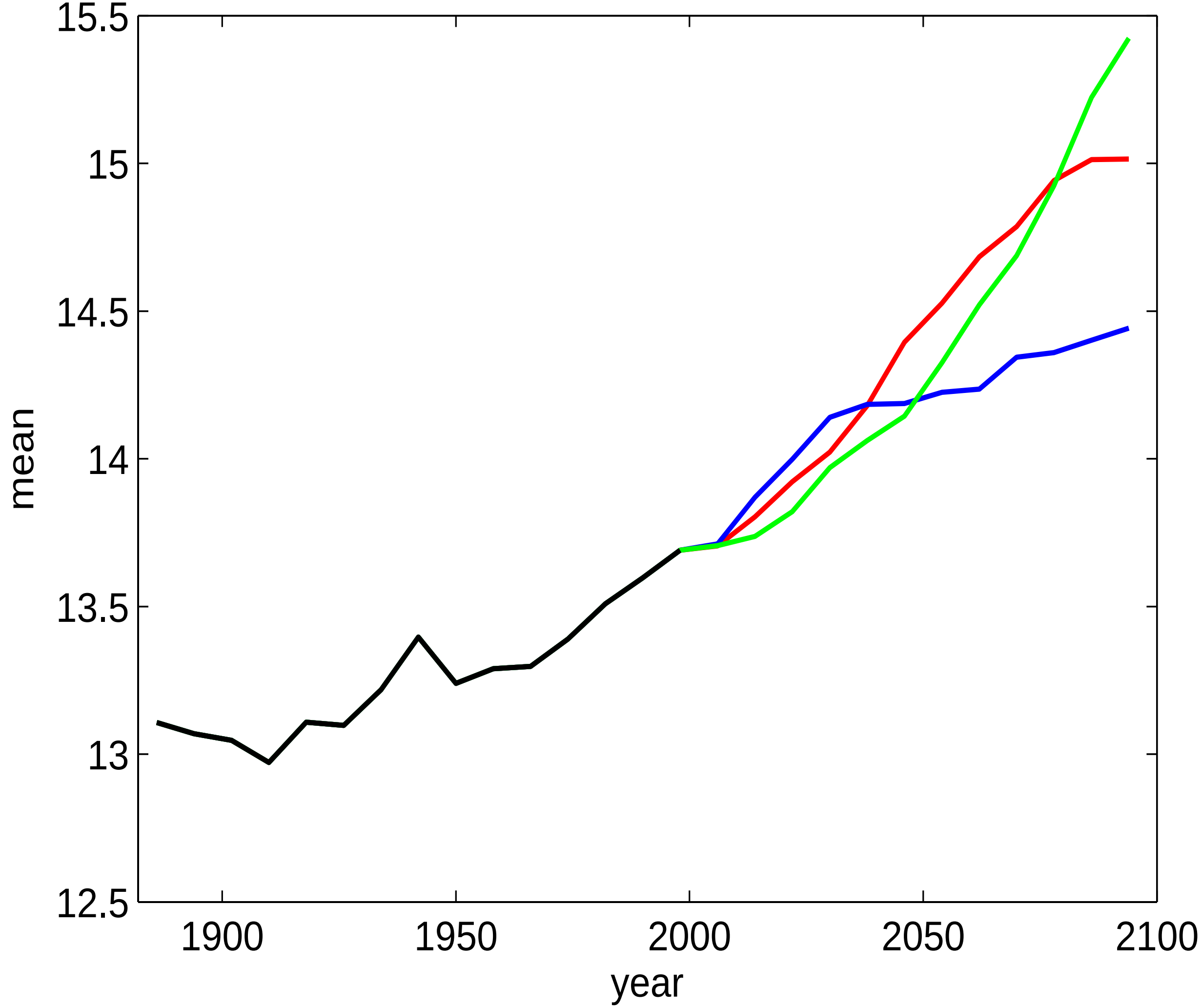
$$\mathbf{X}_{\mathrm{t}} = \boldsymbol{\alpha}_{\mathrm{i(t)}} + \begin{pmatrix} \boldsymbol{\eta}^{\mathrm{n}}_{\mathrm{j(t)}} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\eta}^{\mathrm{s}}_{\mathrm{j(t)}} \end{pmatrix} (\mathbf{X}_{\mathrm{t-1}} - \boldsymbol{\alpha}_{\mathrm{i(t-1)}}) + \mathbf{e}_{(\mathrm{t})}$$

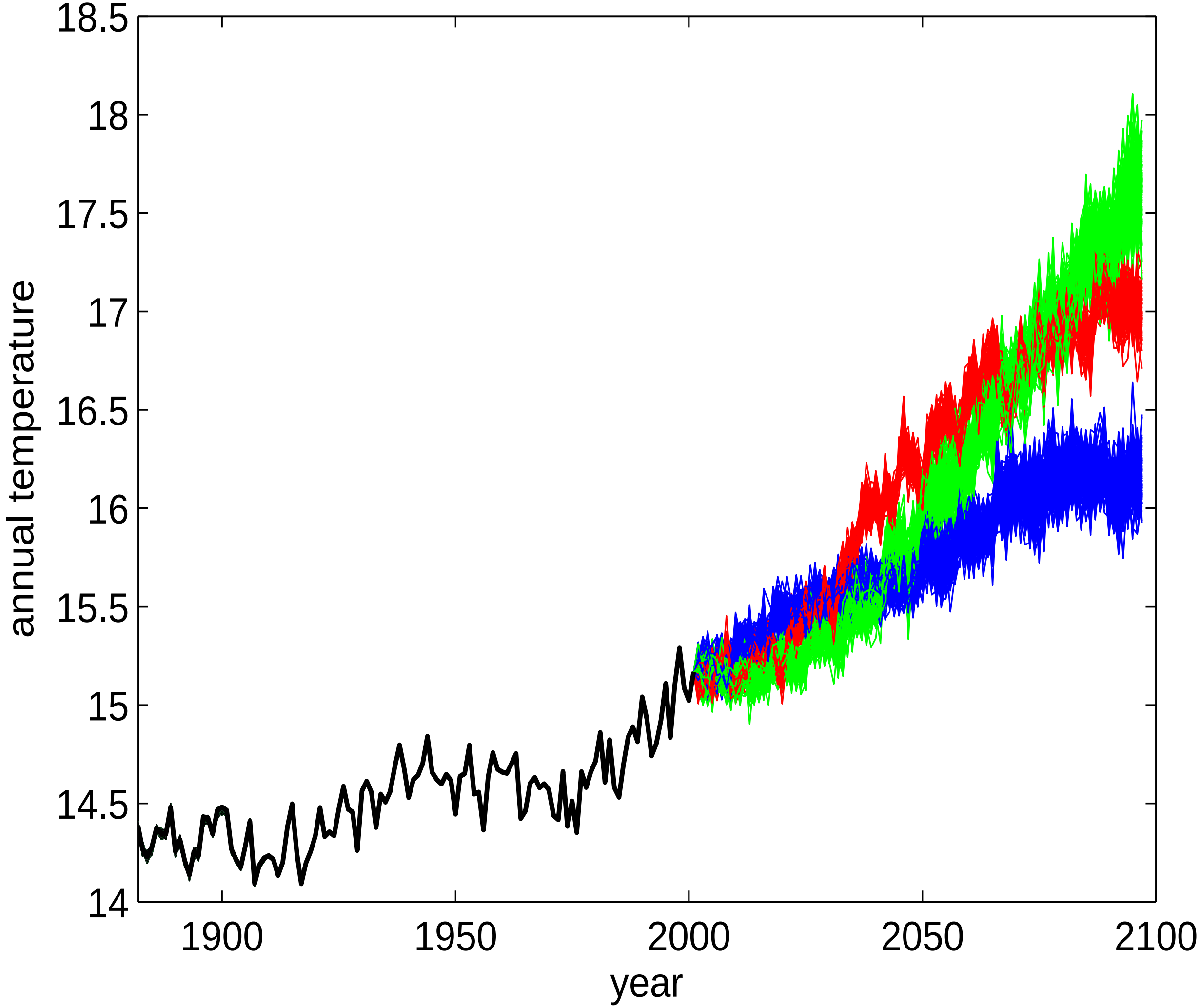
- **Correlated errors**

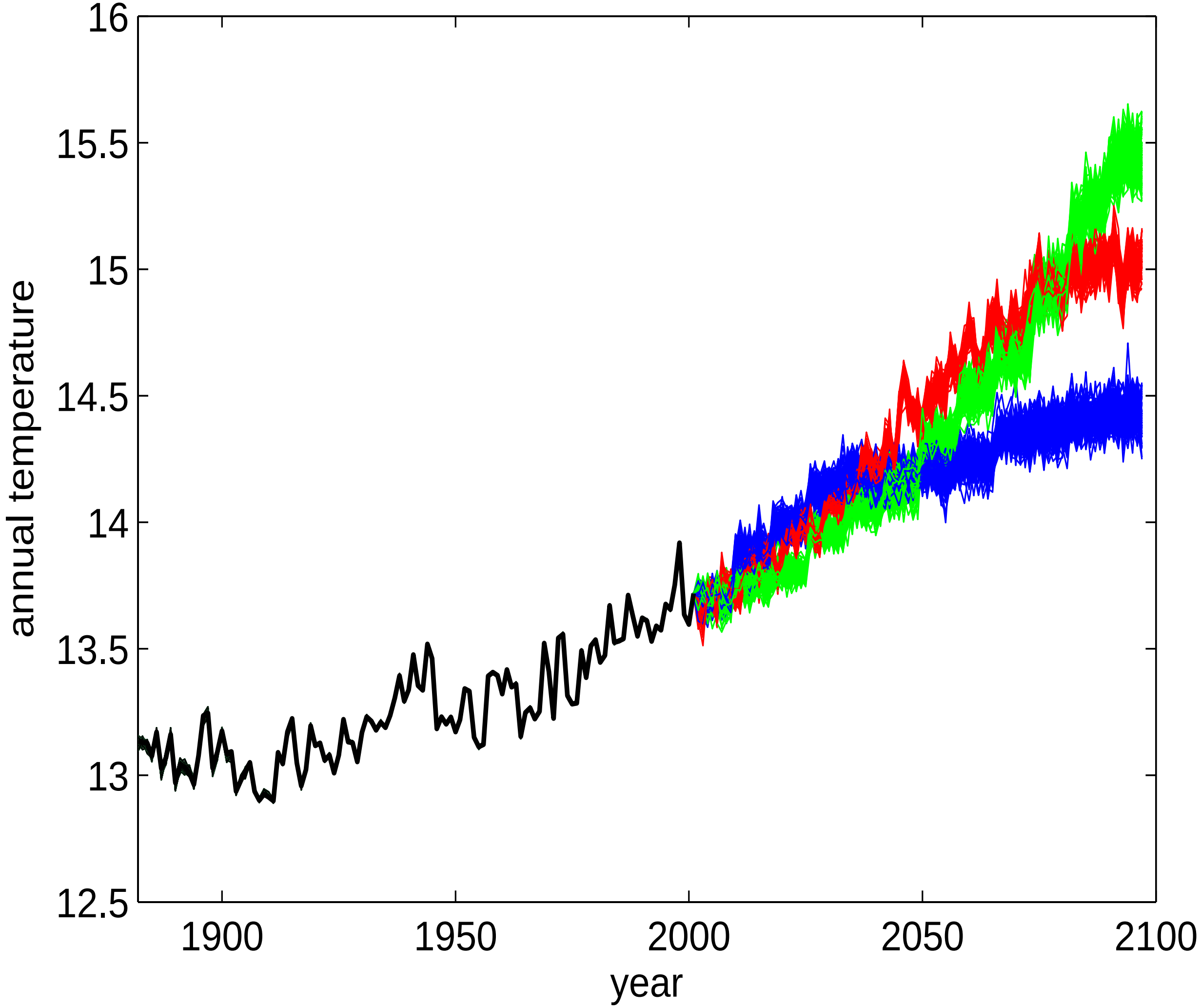
3. $[\boldsymbol{\theta}]$

- **Time evolution:** $\boldsymbol{\alpha}_{\mathrm{i(t)}}$ **slow;** $\boldsymbol{\eta}_{\mathrm{j(t)}}$ **moderate;** $\mathrm{e_t}$ **fast**
  (but variances of $\mathrm{e_t}$ **slow)**
- $\boldsymbol{\alpha}_{\mathrm{i}} = \mathrm{a + b\ CO_{2i} + noise}$
- **Obs period:** $\boldsymbol{\eta}_{\mathrm{j}} = \mathrm{c + d\ SOI_j + noise}$
  Fore period: AR model (i.e., SOI not observed)
- **Variances of** $\mathrm{e_t}$**: AR-like**
- **Model selection!!!**
  Used Obs period data only: slow: 8 years; moderate: two years

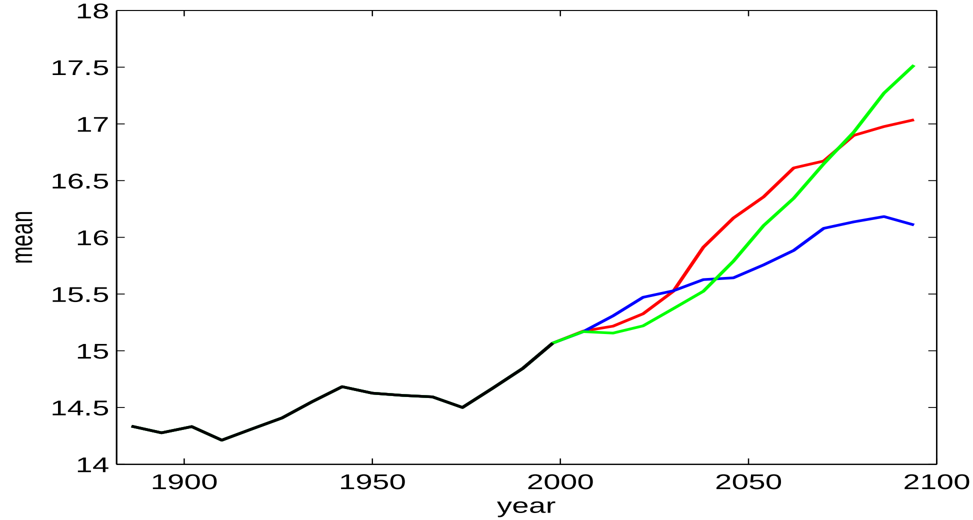

18
17.5
17
16.5
16
15.5
15
14.5
14
mean
1900
1950
2000
2050
2100
year

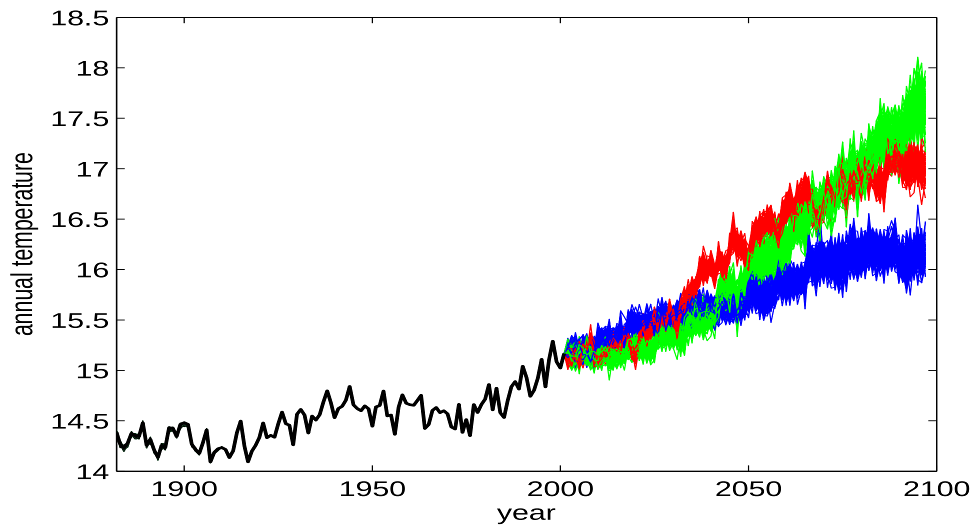

15.5
15
14.5
14
13.5
13
12.5
1900
1950
2000
2050
2100
year
mean

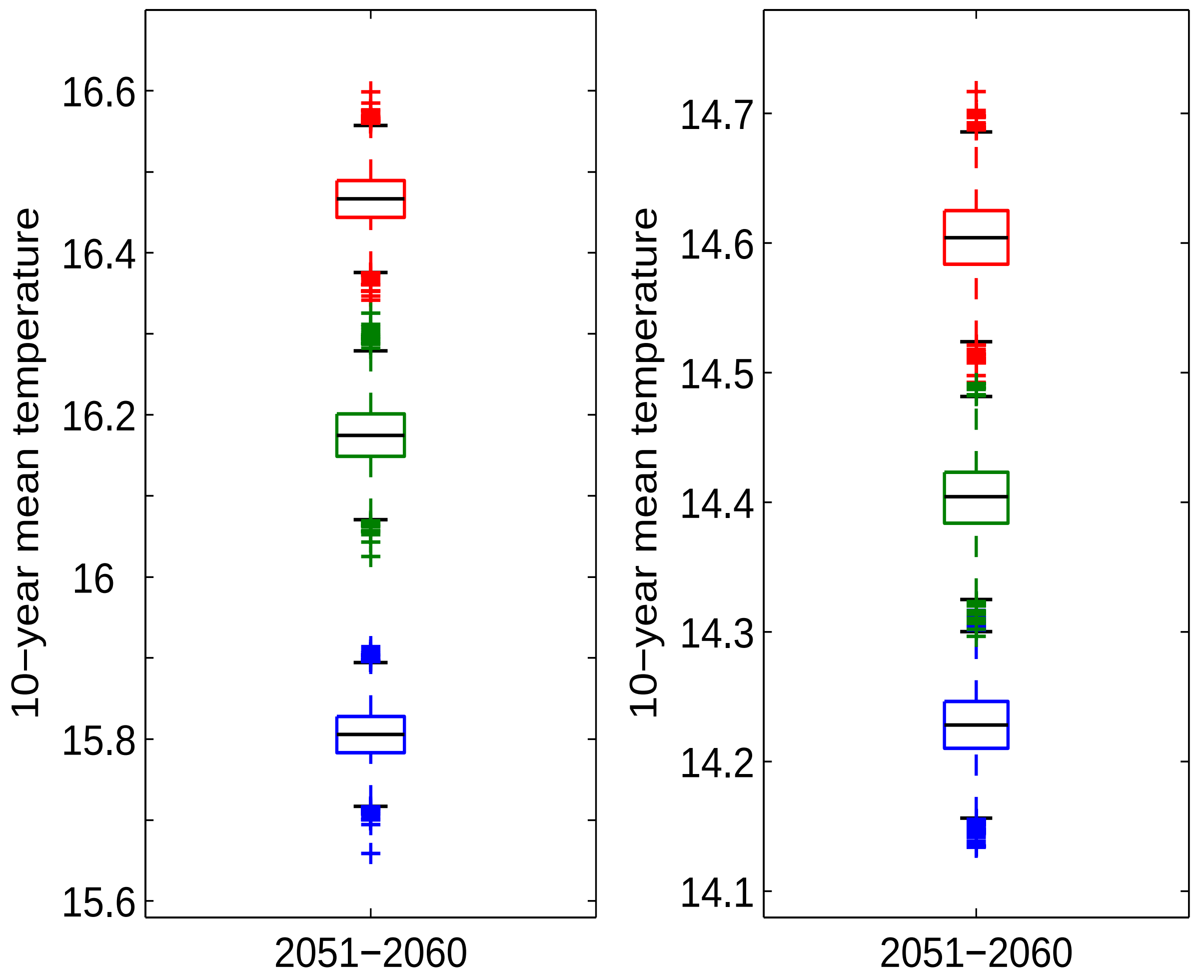

18.5
18
17.5
17
16.5
16
15.5
15
14.5
14
annual temperature
1900
1950
2000
2050
2100
year



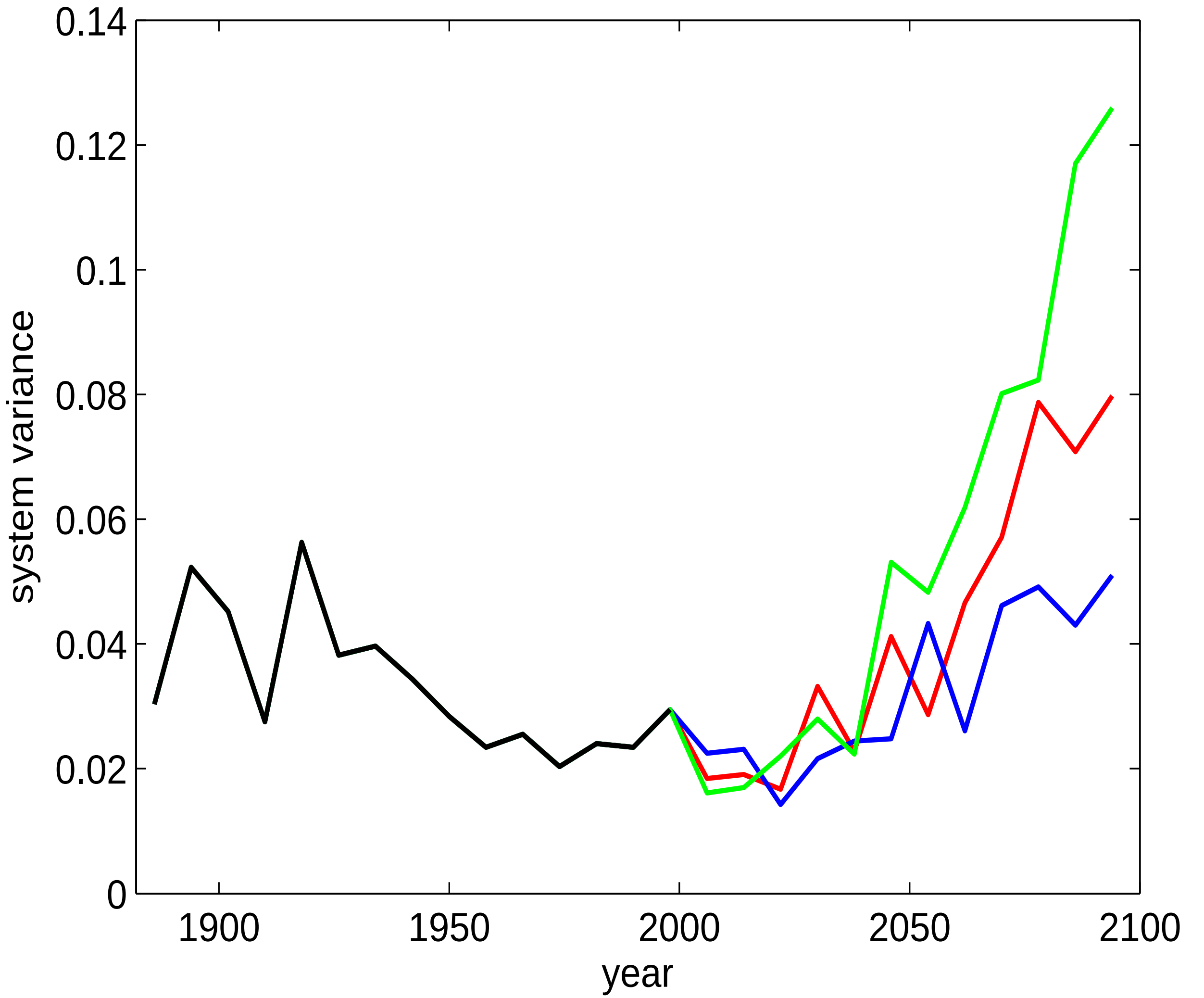
16
15.5
15
14.5
14
13.5
13
12.5
annual temperature
1900
1950
2000
2050
2100
year

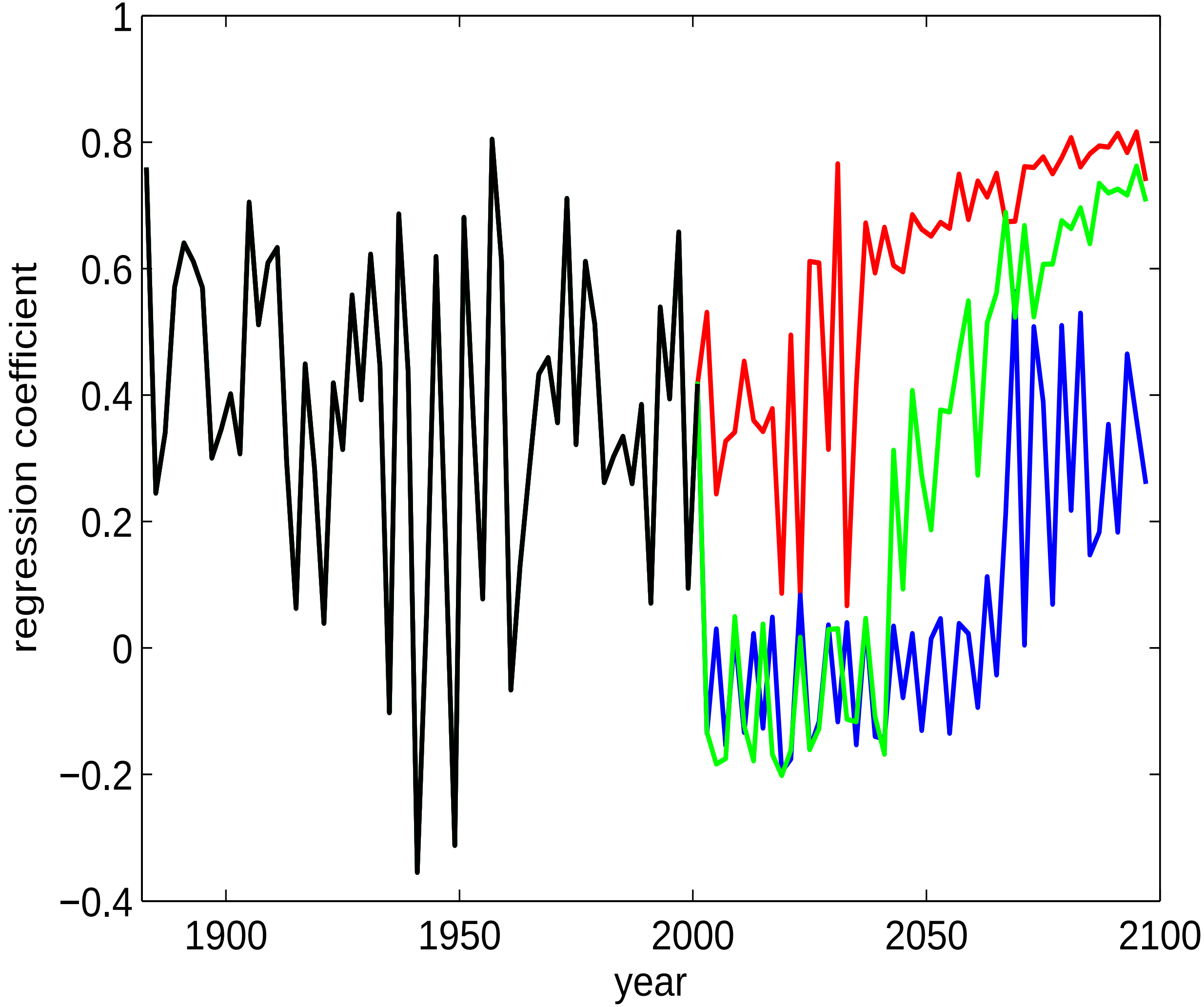

18
17.5
17
16.5
16
15.5
15
14.5
14
mean
1900
1950
2000
2050
2100
year


18.5
18
17.5
17
16.5
16
15.5
15
14.5
14
annual temperature
1900
1950
2000
2050
2100
year



10−year mean temperature
16.6
16.4
16.2
16
15.8
15.6
2051−2060
10−year mean temperature
14.7
14.6
14.5
14.4
14.3
14.2
14.1
2051−2060


0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
system variance
1900
1950
2000
2050
2100
year


1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
1900
1950
2000
2050
2100
year
regression coefficient

Remarks

- **Relax simplifications; assess model.**
- **Information on covariances: climate model assessment; overlap real observations and model output.**
- **Prior on biases in fore. period are crucial**
- **Model classes: Model different β's**
  - **Combine huge (expensive) & simple (cheap) models**
  - **One model with different parameterizations**
- **Dimension reduction: selection of climate variables**
- **Picking the models to use as data vrs the prior on X**
- **Uninformative priors for X**

Discussion

- **Combine observations and model output**
  Wikle et al. (2001) JASA, Hoar et al. (2003) JCGS
  - **Spatio-temporal tropical ocean winds**
  - **Model: features of linearized PDE & a bit of turbulence**
  - **Data: Scatterometer & "Analysis Fields"**
- **Simple models in conjunction with BHM may be better than either "more faithful models" or "statistical models"**